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ABSTRACT

This study investigates the development of mathematical ability and understandings and explores the use of math games as part of a global model of preschool program evaluation. Eighty-three children (aged 3 3/4 to 4 years) enrolled in several programs in the Mt. Druitt Early Childhood Project, North Ryde, Australia, participated in the study. Researcher-designed games, derived from Piaget and Bruner and involving equivalence, seriation, numeration and spatial arrangement tasks, were used in two separate assessments. Statistical analyses of the data (Multivariate Analysis of Variance and other procedures) were conducted. Results indicated few consistent changes over the year from pre- to post-test. Program differences in the most difficult seriation task were found. While counting and number recognition improved generally over the year, overall performance in numeration was inconsistent. Strategies of approach to the number tasks changed over time. Ability to copy spatial arrangements was greater in the post-test, while the difficulty of some prediction tasks remained and in some cases increased. There were significant program differences in the spatial transformation scores. It is suggested that clinical observations of mathematical game activities of preschool children can be useful in assessing curriculum effectiveness. (Author/RH)

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DISCUSSION PAPER

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Evaluation of

Mathematical Thinking

Muriel Healey

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By and large the evaluation of the mathematical abilities of preschool aged children has largely been the province of clinically oriented research studies. Few major evaluation studies have attempted to incorporate the practices inherent in neo-Piagetian mathematical studies as part of their overall evaluation design. The studies reported in this paper represent an attempt to integrate a developmental evaluation approach within the structure of a global evaluation model of preschool education.

John Braithwaite
Field Director
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Evaluation of Mathematical Thinking

1. INTRODUCTION

The evaluation of early childhood programs is beset with problems which arise from the age of the children and the difficulty of selecting any predictive indices for child development. Generally evaluators rely upon standardised measures to determine program effects and often these reflect only part of the change taking place as the result of any interventions. Differences between programs may be there but the more global evidence collected by the standard measures cannot uncover them.

Although it is easy to justify the use of standardised tests of mathematical abilities in terms of reliability, it seems worthwhile also to try and develop some means of recording the pattern of children's behaviour with regard to logical or mathematical problems. Some of the justification for attempting such a measure is the possibility of obtaining feedback for teachers and curriculum advisers about their own curricula. Spreading a net wide enough to cover numerous minutiae may also result in the capture of some larger fish such as the finding of relationships between specific strategies and successful outcomes.

Although the subjects fall within the period preceding "operational thought", that is their thought processes are under the influence of their perceptions, they are capable of working intuitively on various problems. In fact, much of the interest in Piaget's work in recent years has been to discover whether variations in the stimulus materials and in the testing procedures can bring about the solution of "logical" problems at an earlier age (Siegel, 1972; Nigl and Fishbein, 1974; Borke, 1975; Odom, Astor and Cunningham, 1975; Miller, 1976), and to identify the critical experiences upon which operational thought is

constructed (Bearison, 1969; Brainerd, 1973).

Within various preschool programs different instructional emphases are present. For example, Kamii (Bloom, 1971) distinguishes between experiences provided in preschool programs having a cognitive curriculum and other programs. In a cognitive curriculum the child works on problems generated in his play, with the help of the resource adult when required. He is a junior scientist who forms and tests hypotheses, makes predictions and learns from the results of his own actions. In another preschool program the experiences may be planned to follow a route which is developed from the logic intrinsic to the topic, a teacher directed learning. Many standardised tests reflect this "natural" or logical order but it may not be natural or logical to the child. Yet another preschool curriculum will not suggest any specific cognitive outcomes at all. Frequently preschool programs are evaluated by measures reflecting approaches at variance with program objectives. Therefore an approach based on the utilisation of mathematical "games" was adopted to evaluate the cognitive development of the children of the various programs in the Mt. Druitt Early Childhood Project.

The purpose of designing a set of mathematical games for inclusion in the program evaluation measures is to permit the study of the development of concepts having a wider application that the subtests of the Preschool Inventory. The games are derived from the child development literature and in particular from the work of Jean Piaget (1952, 1964) and of Jerome Bruner (1966).

Mathematical problems are situations with real objects which call for some investigation and which can provide solutions of mathematical significance. A "good" mathematical problem is not only interesting but

also requires the child's action for its solution. It should be possible to solve at a number of different levels, and also to be translated into a number of different physical embodiments (Nelson and Kirkpatrick, 1975). The mathematical significance of events is not necessarily expressed in numerical form. However, there is a large number of daily situations which call for numerical interpretation and many of these fall within the scope of measurement. Measurement is applied to any situation in which quantification is required, since it assigns numerical value to objects on the basis of some common attribute (Robinson, Mahaney and Nelson, 1976). Other mathematical interpretations reflect relations between events, relations like equivalence, order and congruence.

Some properties of the real world are easier to abstract from reality than others. Discrete objects can be counted quite easily, for instance, but their spatial arrangement is something which cannot be expressed symbolically in many cases. Piaget has proposed that there are at least two major categories of operational or logical thought. One is logico-mathematical, which is based on deductive inference, and the other is infra-logical and is concerned with imagery, or representations of real experience. The latter may include perceptions of actions, states and transformations received through each of the sensory modalities. Number and measurement fall in the first category although they can be represented by imagery, but order has both mathematical and spatial properties. Hence it may require the support of infra-logical operations derived from imagery. Any problem requiring physical transformations of the stimulus materials may incorporate imagery into its solution (Piaget and Inhelder, 1971).

The play of young children is the source of experiences which provide the necessary imagery. Children transform materials and their actions are

coordinated with their perceptions to produce memory images which may evoke appropriate actions when the situation is encountered again. Play is also one means for testing predictions for it permits experiment and provides consequences against which predictions can be checked. However, it would be a mistake to attribute precision or objectivity to mathematical thinking at this age. It is intuitive and partial and tends to be isolated from other thought. Thus the child may reason logically with one set of objects but fail to apply this reasoning to another.

Some researchers would disagree with Piaget's finding that the logical understanding of number begins when a child develops a sense of logical necessity of the concept of invariance (Piaget, 1952). Susan Carey is one who believes that the mastery of any concept is specific to the situation and to the complexity of the data (1974). Some of her observations are pertinent to this study because they emphasise the socio-economic and personality factors which, in addition to cognitive skills, are implicated in the lack of development of a sense of efficacy and goal setting behaviour. While cognitive powerlessness can be modified by teaching methods which undertake to break each skill into parts and reduce the "degrees of freedom" of the learning situation, the socio-personal aspects of mastery can not in fact be taught. Nevertheless, it is likely that there is an interaction between specific task mastery and the general sense of competency in that success on specific tasks may lead to a modification of a negative self image and thereby increase the child's willingness to set goals for himself.

For the most part the tasks though derived from Piaget's theoretical works are embodied in games of intrinsic interest to preschool children. Toys were attractively made and brightly coloured and the experimenter attempted to keep a play atmosphere by introducing novel items and by

moving through the tasks with as much animation and enthusiasm as possible. Verbal ambiguity was reduced where possible by attempting to reach the idiosyncratic forms used by the children. Some of the emphases of the neo-Piagetian literature have been adopted. For instance, it has been the objective of these experiments to discover the perceptual preferences of the subjects, the effects on response behaviour of complexity within the perceptual field, and the capacities of young children to impose structure upon the perceptual field or to isolate certain features of it. It was not expected that consistent patterns of behaviour would necessarily be shown but that a picture should emerge from these observations of the developing capacities of preschool children to set goals and accomplish objectives in play.

The tasks included in the mathematical games incorporate four areas adjudged to be critical in the formation of preschool aged children's mathematical abilities. The tasks are:

Equivalence or the classification of objects according to children's idiosyncratic perceptions;

Seriation or the recognition of similarities or differences of length or height and the ordering of objects according to the perceived properties;

Numeration or the quantification of objects or events in time;

Spatial arrangement, the imitation of spatial order, and memory of spatial order after various transformations.

Subjects

The subjects were 83 children who were enrolled in the Mt. Druitt Early Childhood Project. They were 3 years 9 months to 4 years at the beginning of the year and they were tested twice, once in Term I and once in Term III of 1977.

Subsequent sections discuss the tasks employed in the four areas and present the results of the children on the tasks. Significant results are reported on a criterion of $p<.05$.

2. EQUIVALENCE

The process of abstracting the common properties of objects or events is a phenomenological one. It is the process of acquiring meaning from a motley array of perceptions and thus finding order in reality. The order observed is at first expressed in actions, then in words, and finally it may be represented numerically for instance by the addition and subtraction of sets. Bruner (1966) describes the ways in which children of different ages and different cultures group objects, and the verbal justification they give for their groupings. Children of six years and older use several different kinds of syntax in grouping (e.g. "they are all round" which is a superordinate statement) and they base the groups on different sorts of properties. Such properties supply the meaning or the semantic aspect of the grouping behaviour. Typically young children associate objects according to perceptual features: for instance, a paint brush and a pencil may be grouped because they are both long rather than because their functions are associated. However, this egocentric preoccupation with the look or the feel of things is later replaced to some degree by association through function. For instance, a spade and a trowel are now grouped because they are used in the work which is done in the garden. Bruner argues that the increasing abstraction of meaning from the concrete perceptible features occurs as the result of this decentering through the consideration of the function of objects and that finally the "intrinsic" function bears no resemblance to the perceptual features which at first held the child's attention. The spade and the trowel may then be grouped as garden implements, the nominal category describing the function.

The spatial or configurative aspect of categorisation is emphasised by Piaget and Inhelder (1964) in their studies of classification using geometric shapes. They find that the youngest children make patterns from the shapes because they have not abstracted the intrinsic properties of the possible classes of blocks. A higher order of classification, forming a hierarchy of classes, requires the child to combine the intrinsic and the extrinsic features of each class of objects. In this context intrinsic refers to the attribute distinguishing each member of a class and extrinsic refers to the whole or exhaustive category "all of the red ones" for instance. We did not expect our young subjects to form such exhaustive categories.

2.1 Aim

The aim was to discover whether the preschool children could group various kinds of objects according to some common property, and what reasons they might give for grouping them.

2.2 Method

There were three collections of objects from which equivalence classes could be drawn. The first collection was a box of small toys which would be familiar to most of the children; the second was a box of environmental objects; and the third was a box of small geometric shapes of different sorts, colours and thicknesses (See Figures 1, 2, 3).

The boxes were presented in the order referred to above. The child was asked to examine the objects and to indicate whether there were any he* did not recognise (based on a procedure designed by Huntsman

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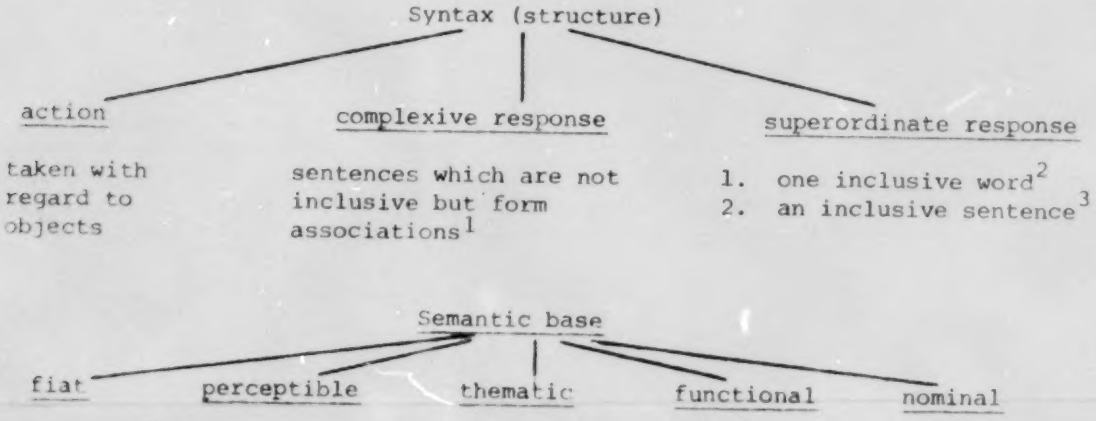
* In the ensuing procedural description the subject whether male or female shall be designated "he" and the tester "she" for ease of reading.

1974). The necessary information was provided. Then the child was asked to put in one place the things which were "a little bit the same" or "those which go together". He was given an opportunity to add more items to the list and if he had finished he was asked to say why he had chosen those things or why they went together. Actions were accepted as explanations if no words were forthcoming. Three trials from each box were given before moving on to the next. The toys were always presented first because it was believed that ready made phenomenological categories might exist for these. The shapes were given last. Records were made of the number and type of selections as well as the reasons given for the selection.

The responses were interpreted on a reduced scale from the one Bruner describes (1966: 71-77). This is because our subjects are younger and less articulate. When actions are the only response they are recorded as justification for the grouping and given interpretations in line with Bruner's categories of verbal explanations. Bruner interprets the explanations in two ways according to their syntax, or structure, and also to their semantic base, or meaning. The reduced categories for the purpose of this experiment can be expressed diagrammatically as follows:



[1] example: the child chooses an envelope and a postcard. "That's a letter but this one has a picture on it." (mixed base)

[2] example: "triangles" (with nominal base)

[3] example: "They for building a house" (with functional base)

While there is a clear hierarchical order for syntactic interpretation, the semantic base cannot be evaluated in this way, although Bruner claims that fiat responses are the least useful and that reasons based on perception emerge earlier than other sorts of semantic responses. Bruner does not include the thematic response in his list of semantic categories, because he claims that it has a typical syntactic structure based on the use of conjunctions. Our interpretation is based on the meaning given by various actions of the child (actions which could be considered divergent) or from verbal story-type responses.

2.3 Analysis

The data were analysed according to the nature of the observations. Interval data, the number of objects chosen in each trial, were analysed by Multivariate Analysis of Variance (Clyde, 1969), and the nominal data were grouped in a series of contingency tables (NUCROS Macquarie, 1969) on which a chi square value could be computed.

Results and Discussion

The only interval data available for analysis by MANOVA are the number of items selected. Variations in number chosen over the three types of materials are of interest but comparisons between the numbers chosen in the pre- and post-tests are invalid since testers in these two situations interpreted differently the amount of prompting to add more items. The most common number was a pair of items, but this was usually exceeded in the selection of geometric figures. More children made exhaustive groups in this subtask than in either of the others. It would seem that the materials most likely to be grouped in pairs were the environmental objects (see Table 1) and that this tendency resisted prompting.

Table 1: Number of Occasions in which Pairs were Selected

	Pre-test Range over three trials	Post-test Range over three trials
Toys	45-60	65-68
Objects	65-68	67-72
Shapes	9-11	18-22

There was no significant relationship between the number selected and the reason given for grouping, but the figures in Table 1 suggest that readily available perceptible features facilitate the formation of larger groups, while smaller groups are formed when properties have to be "abstracted" or invented. However, the few children who did group larger number of toys tended to use a superordinate sentence or a "one-name" superordinate, while pairs of toys were described in complexive syntax half as often as they were grouped by superordinates. In the post-test there were some differences site by site in the numbers of items chosen ($p<.033$) and the greatest difference was in the first trial of environmental objects. Here the greatest mean number was chosen by the children in the Contemporary Program. This result suggests that the children of this program were not as inhibited as the remainder of the subjects by the less obvious relationships between the environmental objects.

Chi square tests between the syntax and the semantics of the grouping explanation show a significant relation in nearly every case (see Table 2). However, the relationship cannot be interpreted simply as an association of higher order or lower order responses. Superordinate responses are considered the most inclusive, exhaustive and explanatory, and therefore they can be regarded as a basis for the addition and subtraction of classes. Complexive structure, on the other hand, permits a greater flexibility of association. Whereas this may not be useful for such logico-mathematical operations at this age, it may serve to provide a much wider degree of transferability of concepts and relations between classes at a later point in the child's development.

In this study there are three types of description for the syntax, or structure, of the equivalence classes (see p. 8). There are five types

Table 2: Number of Responses in the Pre- and Post-tests Classified by Syntactic Structure and Semantic Base

	Trial	Test	Syntax				Semantics*				Signif. of Chi Square Relation
			Action	Complex	Name Super	Gen Super	Fiat	Perceptible	Functional	Nominal	
TOYS	1a	Pre	8	9	24	42	7	16	11	47	.001
		Post	1	22	26	34	5	16	15	47	NS
	1b	Pre	15	5	23	39	11	29	8	34	.001
		Post	3	21	23	36	7	20	19	36	.02
	1c	Pre	9	12	24	37	9	23	8	39	.001
		Post	7	25	12	39	5	21	16	40	.001
OBJECTS	2a	Pre	10	30	19	24	9	22	26	26	.001
		Post	9	25	19	30	8	18	21	35	.001
	2b	Pre	18	24	12	29	9	29	23	21	.001
		Post	8	23	16	36	8	28	17	30	.01
	2c	Pre	19	24	16	21	14	28	19	20	.001
		Post	6	18	16	42	11	29	16	26	.001
SHAPES	3a	Pre	19	6	30	27	11	47	1	24	.001
		Post	4	5	28	46	6	50	2	25	.01
	3b	Pre	17	5	30	30	9	48	0	26	.001
		Post	4	6	27	46	4	50	1	28	.001
	3c	Pre	21	4	24	33	11	47	1	24	.001
		Post	3	4	30	46	6	45	3	29	.01

* Thematic omitted because N too small

of semantic modes on which these structures are based; fiat, perceptible, functional, nominal and thematic (divergent, or contextualised within a story). The prevalence of nominal responses to the toy stimulus materials is associated with superordinate grouping of either the one-word or sentential variety. Names are often allocated without an inclusive sentence, but the gesture accompanying the name suggests that all of the articles are being named. Functional responses are more numerous when environmental objects are grouped and the supportive syntax is more often complexive than for any other stimulus materials. Perhaps this is due to the greater difficulty in forming equivalence between items which are not customarily grouped, like the toys, or perceptually similar, like the shapes. Not surprisingly, functional bases for grouping are infrequent when geometric shapes are selected, while one-name superordinates are very common, the name being either a noun or an adjective (e.g. green). The use of nominal grouping requires language, but functions, and to a lesser degree perceptions, may be expressed in action. Some of these more primitive responses recall the graphic collections described by Piaget (Inhelder and Piaget, 1964).

Although the nature of the stimulus materials clearly has some effect on response styles, the majority of the syntactic structures are superordinate (given the benefit of linguistic omissions) and the majority of these refer to a name or a perceptual feature. The relationship is very clear where the geometric shapes are concerned (see Table 2). However, when one word is used to classify these shapes it is sometimes difficult to decide upon the interpretation of the semantic base. It was decided to group all adjectives into the semantic category "perceptible", while all nouns were interpreted as names, "nominal". Thus all colour words and shape descriptors like "round" are categorised as perceptible, while "square" is given nominal status along with "triangle" and "circle".

Almost all of the responses to geometric shapes are in these categories, and are described by one word with an inclusive gesture.

At the end of the year subjects are more articulate as exemplified by the decrease in number of action responses, but one word superordinates are still common. There is an increase of complexive responses to toys and of complete sentence superordinates to the other stimulus materials. However, the general pattern of responses remains the same at the end of the year as at the beginning of the year. There are more superordinate groupings of environmental objects and geometric shapes, and this is associated with an increase of nominal bases for grouping; but these are only trends. The greater availability of names for groups may reflect a growth of vocabulary rather than a change in grouping strategy, but the ability to economise on class descriptors in terms of one word explanations may be of advantage in mathematics. At this stage it is not certain that naming a class of objects in a superordinate reflects an understanding of the extensive properties of the class. Rather, either noun or adjective serves to discriminate a perceptual or other feature of several objects. Because such features are readily available in the geometric shapes, these are the easiest for the young subjects to group. However, when the materials are less structured, as in the environmental materials, even the economical noun is used in complexive syntax, e.g. "That's a brush and that's some paint." Generally speaking there are no differences between programs but there is a trend in the post-test for children in the Cognitive and Contemporary programs to use more complete sentences when making superordinates. This may simply reflect a greater ease in talking to adults.

When objects are handled they acquire a meaning derived from common use which may be different from the conceptualisation of the heard name or the visual attribute. It may however, be harder to put into words. Many of the actions associated with grouping are clearly functional even if the verbal response reflects perceptual meaning. This suggests that more than one grouping is possible for a child but one may be expressed more readily than another. Movements or actions are more difficult to quantify than discrete objects, or to express in the mathematical sentence "They are all _____" (x = y). Also the necessity to respond to the question "Why are they the same?" or "Why do they go together?" may distract from the conceptualisation responsible for the action. Quite a number of subjects refer to perceptual similarities of the paint brush and the paint tin while actually making painting motions. The necessity to justify similarity requires that some quantifiable property be abstracted from the objects. If the only thing in common is a function this may be difficult for the child to define as a common attribute of a number of objects, unless familiarity through play has made it more obvious.

Another example of mixed response is one in which the subject placed the geometric shapes on the floor in a pattern reminiscent of Piaget's graphic collections (configurations) but then referred to the colour or the shape of all of the blocks. The facility with which most children abstract a single descriptor (colour or shape) suggests a learned response which may begin at the mother's knee. Those preschool programs emphasising this kind of visual discrimination are reinforcing such a pattern.

Whether this kind of grouping is the most useful for future conceptualisation is open to question. In fact, the loss of the complexive sentence response, which links unlike items by association, may be a trend in the direction of mathematical parsimony (named sets), but it may also indicate a loss of creativity which can be found in flexible styles of grouping. While logical or mathematical activities may benefit from the uniformity of meaning in nominal superordinate groups (or form classes as Bruner calls them) language fluency and creative expression may not. The functional mode is in the happy position of being equally adaptable to sentences telling a tale or creating an association and to functional categories which can be given a label.

3. SERIATION

Although topological order is perceived fairly early in life (Piaget and Inhelder, 1956) seriation, or systematic ordering, is a later acquisition because it incorporates mathematical properties into the spatial configuration. Order relations within a series are transitive: weight B is heavier than weight A but is also lighter than weight C. When serial correspondence is attempted a qualitative correspondence between the elements of each series is required. According to Piaget this does not require anything more than is involved in the formation of the single series. The spatial configuration supports the correspondence. If the elements are placed in partial correspondence with one another the qualitative relationship is obvious. However, when either series is disarranged, an understanding of ordination is necessary to re-establish the corresponence (Piaget, 1952).

Kamii (In Bloom, 1971) describes three stages in the development of seriation. First, there is random placement of the items, second

perceptual seriation, wherein the configuration of the series rather than the logical relation dominates the process and third logical seriation, which is systematic. Since there is a configurational component of any object series, some objects may lend themselves more readily to seriation than others. Size may be a less salient organisational clue than colour or shape (Mackay, Fraser and Ross, 1970). However, training can increase the salience of the dimensions of a series (Odom, Astor and Cunningham, 1975) but this may affect the recognition of a series more than it affects the reconstruction of a series (Blackstock and King, 1973). Other factors influencing the capacity of the young child to construct a series are the number of items and the dimensional differences between them (Siegel, 1971, Siemon, 1978).

3.1 Aim

The aim was to discover whether the children could

1) discriminate between objects of different lengths;

2) reconstruct a series with strong configurational properties;

3) construct a small and a large series spontaneously, or with assistance;

4) recognise the qualitative correspondence of elements after displacement.

Tasks 1 and 2 were practice sessions in which it was revealed whether the child would have any difficulty with the necessary discriminations, and which would, it was hoped, provide a set for the main task of constructing a series.

3.2 Method

Apparatus: The task of seriation was thus approached in a series of steps. First the child was offered an envelope holding various cut

drinking straws and some plastic string. Some of these were equivalent in length and some were not.

Next the child was asked to examine an arrangement of wooden beads on four upright pieces of dowelling mounted on a base. The heights of the dowel rods varied systematically. On the longest dowel were four red beads, next three yellow beads, then two blue beads and finally one green bead (see Figure 4).

The apparatus for the main seriation task consisted of two sets of six blocks varying in height by 2cm. On each was a face at one end. On one set of blocks the faces were of one sort ("gollywog" faces), on the other another sort ("smiley" faces) so that each could be identified as belonging to the one family. These two sets when combined formed a larger series by the interweaving of the two families. The dimensional differences were thus halved (1cm) and the total N of the third series was thus 12 (from 5cm to 16cm in height) (see Figure 5).

3.2.1 Length discrimination

The envelope was given to the child and he was asked to pick out a straw which was longer than one held by the tester. If doubtful the tester said "find the longest straw". Then the child was asked to find a straw "as long as", "the same size as" the standard. Finally he was asked to find a string which was "as long as the straw".

3.2.2 Warm-up session

The arrangement was shown to the child and he was asked to examine it and to describe it before removing the beads from the dowels. His

description and actions with regard to the toy were recorded. He was told that he would need to remember where the beads belonged because he would have to replace them after they were removed and they were to go back where they were before. The order of replacement was recorded as well as its accuracy and whether the child was able to correct any errors. These two games were intended to draw the child's attention to the dimension of length or height and to provide practise in making comparisons.

3.2.3 Construction of series

The box of 12 blocks was handed to the child and he was asked to select one of the families to play with. After selection of the family and of the father (the biggest), and the baby (the smallest), he was shown where the father and the baby would stand so that the rest could go in between with the tall ones near the father and the small ones nearer the baby. The child was given the opportunity to order the blocks spontaneously. If he did not seriate then help was given in positioning the end blocks and, with action to support the instruction, the tester asked the child to arrange the other family members so that they got smaller and smaller down to the baby. If the child did not then seriate the tester provoked the successive action by asking "Why one goes next to this? Is this the right one?" (Series I). When he had finished the child was allowed time to correct after the tester said "Do you think that is right now?" Finally, when a series had been constructed he was asked to describe what it looked like while the tester ran her finger along the top from the tallest to the smallest.

The second 'family' (Series II) was dealt with in much the same way but not before any errors in the first series had been corrected by the

tester. After the second series was formed the child was asked to place the two families near each other (one set in front of the other) and to say whether they were the same or different. The purpose was to elicit an awareness of height discrepancies between the two series.

A game was now played which consisted of lining up the two families for the photographer. Each block was to stand near its friend (Series III), the father next to the father, the mother next to the mother, and so on. Thus it was an interweaving of the two series. To assist in breaking the present configuration the tester placed the taller 'father' at one end at some distance from the other blocks and the smallest 'baby' at the other end and spaced out one of the 'families' so that there was room for the 'friends' from each series to stand next to each other as if they were holding hands. Again the arrangement was provoked step by step if the child failed to seriate spontaneously.

In each case the correctness or otherwise of the child's responses were recorded, and the final arrangement in each case was also recorded, whether the arrangement was spontaneous or provoked and whether the child had chosen the starting blocks (1 and 6) himself.

3.2.4 Ordination

Finally, after forming a complete series by interweaving the two families, the blocks were again separated into the two smaller series, one in front of the other. The child was told that they were going to play the game of 'find the friend'. The family behind (taller) was displaced so that there were spaces between its members while the other family remained in closed file. The tester touched the extreme blocks of the displaced series one after the other saying "Where is the friend of this one?" Then the second and the fifth blocks were touched one

after the other and the same question asked. Finally the question was put after the tester touched the inner blocks of the displaced series one after the other (the third and the fourth).

3.3 Analysis

Again Multivariate Analysis of Variance was used to determine whether there were any program differences in the ability to discriminate length, reconstruct an arrangement, construct series of 6 and of 12 elements and to recognise ordinal relations between elements of two series.

3.4 Results and Discussion

The child who senses that some order is inherent in the materials may begin to place them spontaneously. In the case of the two block 'families' a sense of purpose is shown by many subjects in separating out the members of each family group first of all. Some children also select the tallest spontaneously and a few order the remainder even before instructions are given. Any child who forms a row of blocks in an orderly manner without prompting by the tester is credited with spontaneity even if an error is made. Failure to order is not due to difficulty in discriminating length or height, since over 90% of all subjects are successful in discriminating straws on the instructions "Find me the longest straw", and "Get me the straw that is the same size (or as long as) as this one". There are some children who fail to respond correctly to the instruction "Which string is as long as this straw?" (25% in the pre-test and 18% in the post-test), but this is due to the difficulty experienced in actually equating a flexible with a rigid material.

Table 3: Percentage of Spontaneous and Provoked Responses

Series	Pre-test		Post-test	
	Spontaneous	Provoked	Spontaneous	Provoked
I	36.2	63.8	65.1	34.9
II	43.6	56.4	78.3	20.5
III	6.4	91.5	67.5	32.5

There was a substantial increase in the percentage of children who seriated spontaneously between the pre- and the post-test, as shown in Table 3. Also the second series is ordered spontaneously by more subjects than the first series. This suggests that a large number of children perceive the order inherent in the materials once they examine them and understand the experimenter's instruction. However, the interweaving of the two series of six presents a number of difficulties. First, the dimensional differences between the members is now halved (see p. 18). According to Siemon (1978) the relations between members of a series are more difficult to conceptualise when the real differences are reduced. Also the inner blocks are more difficult to order than the end blocks (Siegel, 1972). In addition to these complexities the actual interweaving task provides a number of distractions. The subject has already achieved a configuration of a series of six in two instances and these have to be broken and interspersed with one another. The separate configurations are reinforced by the concept of family and the similarity of the faces. Since children of this age are bound by already formed percepts (what Bruner calls "perceptual stuckedness"), they need to have a fairly well defined concept of the transitive relations between any two or more elements of the combined series to overcome the family "gestalt". Most of the subjects apparently expect the two families to be equated in size, and the forming up of pairs of friends no doubt assists in this expectation. Another problem was the personification of the 'baby' block. Some children place this next to the mother or in some other central position. No differences were shown between programs in any of the pre-test seriation and ordination responses, but the multivariate test of significance shows a difference between programs in the post-test ($p<.007$). Those items in the seriation-ordination tasks reflecting the program differences are the double seriation task, the total number of

end blocks correctly chosen and two of the three pairs of ordinations (see Appendix A).

Table 4 shows the mean group scores and the standard deviations for each of the sex groupings of the five programs in both the pre- and post-tests. The scores in the double seriation task are more diverse in the post-test because some groups have reduced their mean scores but enlarged their standard deviations (Contemporary and Behaviourist) while others have increased their mean scores and reduced some standard deviations (Cognitive and Competency).

A general increase is seen in the number of end blocks correctly placed in the post-test and also a general reduction in the variance of this score. However the ordination scores do not show the same trend. Their variation is great in both pre- and post-tests but the scores are differently distributed across programs on these two occasions. In comparison with other programs the Behaviourist program fares badly, and the Cognitive program does not exhibit the same competency in ordination as in seriation, compared with other programs. While some reduction in the accuracy of seriation scores from pre- to post-test could have been attributed to variations in prompting techniques, this does not in fact occur as often as one might expect. The same explanation does not in any case apply to ordination which was not provoked in any way. Perhaps some classroom techniques reinforce the configuration of the series rather than its logical properties.

In both pre- and post-tests there is a significant relationship between the degree of confidence the children have about the first seriation tasks and its outcome (based on Chi Square associations derived from the NUCROS program as in the previous experiment). The majority of

Table 4: Group Means and Standard Deviations on Items Showing Program Differences

	Series III		End Blocks		Ordination II		Ordination III	
	Pre Test	Post Test	Pre Test	Post Test	Pre Test	Post Test	Pre Test	Post Test
Cognitive								
Boys	7.71 (4.68)	10.00 (2.83)	5.14 (0.90)	5.71 (0.49)	0.86 (0.90)	0.86 (0.38)	0.57 (0.98)	0.14 (0.38)
Girls	8.50 (3.42)	9.40 (3.40)	5.50 (0.76)	5.50 (0.54)	1.50 (0.93)	1.38 (0.74)	0.75 (0.04)	1.13 (0.99)
Competency								
Boys	8.72 (4.22)	9.71 (2.43)	4.86 (1.57)	5.57 (0.54)	1.43 (0.54)	1.57 (0.54)	0.86 (0.69)	1.00 (0.82)
Girls	10.63 (1.60)	11.13 (1.64)	4.50 (1.31)	5.88 (0.35)	1.38 (0.74)	1.38 (0.92)	1.00 (0.76)	0.75 (0.89)
Contemporary								
Boys	9.29 (2.87)	6.43 (3.31)	4.29 (1.50)	4.57 (0.98)	0.86 (1.07)	1.00 (0.82)	0.43 (0.54)	0.71 (0.95)
Girls	10.86 (1.57)	6.29 (4.39)	5.00 (1.56)	5.43 (0.54)	1.71 (0.49	1.71 (0.49)	1.00 (1.00)	1.71 (0.49)
Behaviourist								
Boys	10.17 (2.23)	8.50 (3.99)	5.00 (1.55)	5.67 (0.52)	0.50 (0.55)	0.83 (0.41)	0.33 (0.52)	0.00 (0.00)
Girls	9.88 (2.19)	6.14 (4.38)	5.43 (0.79)	5.29 (0.76)	1.29 (0.95)	0.71 (0.95)	0.86 (0.90)	0.57 (0.98)
Home-based								
Boys	8.21 (2.70)	8.15 (3.48)	5.21 (1.37)	5.46 (0.66)	0.50 (0.76)	1.39 (0.77)	0.50 (0.52)	1.15 (0.90)
Girls	8.25 (3.84)	9.92 (3.52)	5.17 (0.94)	5.69 (0.48)	0.92 (0.90)	1.46 (0.78)	0.50 (0.80)	1.23 (0.93)

responses to Series II in the post-test were both spontaneous and accurate. The reverse is the case in Series III in the pre-test. The relationship is increased in the post-test for Series III (in the pre-test there were few correct responses that were not provoked). Table 5 illustrates these comments (chi squares are calculated according to the numbers of blocks correctly placed). The degrees of freedom range from 6 to 12 when the range of scores is considered but it is only 1 for the 2x2 contingency table illustrated.

4. THE DEVELOPMENT OF A CONCEPT OF NUMBER

The difference between counting and number correspondence is at its most extreme when children first learn the number names and recite them more or less correctly in serial order. In general experience the first attempt at correspondence is one in which objects are paired off, next when numbers are applied to discrete objects, like toes or toys. The more difficult correspondences are those in which the recognition of the units to be enumerated is not easily accomplished. When length predictions are required for instance, the necessity to partition a whole makes the task more difficult. Even more difficult than this is the quantification of units of time or movement for the young child (Friedman, 1977; Piaget, 1970). As Brainerd has pointed out (1976) children were given no explicit instruction in the logical meanings of numbers even after they could apply them until recent years, and the logic is often difficult to define.

For Piaget (1952) number ability develops contemporaneously with the understanding of the additive properties of classes and the asymmetrical relations between them. Moreover the understanding of the part/whole relation is believed to be essential. One way in which the numerical system differs from classification in general is that its members are governed by the arithmetical relationships between whole

Table 5: Chi Square Relationship between Spontaneity and Accuracy in Seriation

		Pre-test			Post-test		
		Correct	1 or more Incorrect	p<***	Correct	1 or more Incorrect	p<***
Series I	S*	18	14	(.02)	49	5	(.05)
	P**	13	38	.01	20	9	.01
Series II	S	26	9	(.01)	61	4	(NS)
	P	14	34	.001	13	4	.05
Series III	S	5	1	(NS)	32	24	(.05)
	P	25	50	.02	6	21	.01

* S = spontaneous response

** P = provokes response

*** The p value in brackets refers to the whole range of correct scores for block placement. The collapsed 2x2 contingency table yields a lower p in each case

numbers. Reality only approximates this type of regularity and can be better described by rational numbers. However the young child can learn to appreciate arithmetical relations between classes or sets of objects by comparing and describing equalities or inequalities in terms of 'same', 'more' or 'less'. While theoreticians may not agree as to the fundamental elements of arithmetic or the order of their presentation in the ideal curriculum, mathematicians believe that the global or intuitive notions of 'more' and 'less' follow the awareness of the relevance of quantification, and that the next step is the assignation of numbers to objects in such as way that the set of 'more' has a larger number than the set of 'less' (Robinson, Mahaney and Nelson, 1975). In this sense number is almost synonymous with measurement. Brainerd (1976) argues that the concept of number can begin either from cardination or from ordination but that there is some evidence that ordinal meaning is easier to teach than cardinal meaning. However this must depend to some degree on the configurative properties of the objects to be ordered. The importance of perceptual cues for intuitions of number and number conservation is contrasted with the lack of importance of perception for operational thought once established (Silverman, Vanderhorst and Eull, 1976). Perceptual cuing has also improved in classifying (Osler and Drax!, 1977) and seriating (Hock and Ross, 1975).

In a study of notational counting by 3, 4 and 7 year old children, Saxe (1977) found that the level of maturity of children's counting strategies is related to specific task demands. At the pre-quantitative level conflict occurs between spatial arrangement and counting. Saxe found that counting strategies and counting accuracy develop together through one to one serial correspondence between objects counted and the number names allocated to them. This type of counting is a systematic

means to extract numerical information from arrays.

In Piaget's theory the response pattern of the pre-operational child is taken to be inconsistent and lacking logical coherence. In contrast the 'operational' child, that is one who has attained a logical necessity in his explanations about the conservation of quantity, is consistent in his responses. However when materials and approaches are varied there are some interesting results. Susan Carey (1974) found for instance that trick variations in the conservation task (liquid conservation) creates a diversity of explanations by young known conservers compared with adult conservers. She concludes that the 'stage' differences between 3 year olds and 7 year olds is spurious. On the contrary she believes that all children pass through the various stages when acquiring mastery of a particular problem, whatever the age.

Inconsistency develops when the child or adult is attempting to combine several known relevant facts but does not know all the possible relations between them. Two aspects of information processing are therefore indicated:

1. the absolute or maturational information processing capacity;
2. the range of individual skills with regard to any particular problem, skills acquired from "learning new constituents, their modularisation and their concomitant incorporation into larger structured wholes" (Carey, 1974:14).

4.1 Aim

The aim was to discover not only whether the children could count and recognise numbers, but whether they could form some numerical correspondences between sets of objects and also correspond numbers with actions or movements. There were four sections which comprised five tasks: counting, measurement, number correspondence a & b and conservation.

1. counting and predicting numbers of discrete objects and recognising the printed numbers expressed by the child;
2. building two sets of blocks in towers and equating them in height;
3a. division of a set of wooden men by the numbers of seats in a vehicle;
3b. correspondence of number to circular motion;
4. conservation of number.

In all of these tasks an attempt was made to assess the strategies of the child in mastering the materials and to record the focus of his attention.

4.2 Method

The apparatus was different for each of the subtasks and will be described in the subsequent sections.

4.2.1 Counting

The child was given an egg carton big enough to hold 6 wooden blocks of equal size, and no more. He was asked to fill the box with blocks and to count them. A smaller segment of egg carton was provided and the subject was asked to predict, either by selecting the correct number (2) of blocks, or by naming, "How many would fit in this little box?" Then the child was asked to place the blocks in the box to see if he was right.

The subject was then shown a chart with numbers 1 through 9 written upon it. He was asked to point to whichever numbers he had named in the previous tasks (6 and 2 if correct).

4.2.2 Measurement

A box of blocks of the same cubic dimensions (7.5cm) was presented to the child. The tester built a tower of 4 blocks and asked the child to build one alongside "as big as mine, so that it will be the same as mine". After completion the tester asked the child how he knew they were the same or as big as each other. Actions and explanations were recorded.

A small flat wooden tunnel (3.5cm high) (see Figure 6) was placed in front of the child and the tester explained that she would now build her tower on top of it. The child was asked whether he had ever seen a house up on a hill. He was then told that while the tester's tower was on the tunnel, he would build his tower down on the table, using both hands to emphasise the two starting points. "My tower starts up here and yours will start down there, but the towers have to be the same. Yours must be as big as mine." The tunnel was of such a height that the two towers could not be equated in absolute height. The only way the towers could be equated visually was for the child to lift his to the same level. If he did this the tester said, "Yes, that would make it easy to see if they were the same, but I want yours to start down there." The child had to deal with the discrepancy of the upper end points in some way. When he had built his tower he was asked whether they were the same and if he was incorrect he was asked "Is yours as big as mine? Is mine as big as yours? Look, yours starts there and mine starts here. Can you make them the same?" Again reasons were asked and recorded.

4.2.3 Division

The materials consisted of a wooden fire engine having four vacant seats, and twelve firemen (see Figure 7). A box was used to simulate a burning building. After preliminary play the subject was told "The men

are resting now, (they were placed in a box) but while they are sleeping a fire starts over here in this house. How can we get all the firemen to the fire?" The child was restrained from beginning the action until he answered the question. Then the tester said "Show me..." If he placed men on parts of the engine or the ladder he was told "Its a rule that the men must stay in their seats. How many men can fit in the seats?" When the seats were filled he was asked "How many trips will the engine have to make to get all the men to the fire?" If the child failed to empty all of the seats after the journey he was encouraged to do so, and although some play was permitted he was reminded of his goal, the goal of getting all of the men to the fire. After the trips were completed he was asked how many trips were made. If necessary a 'trip' was demonstrated with the empty engine. If needed the child might repeat the action by returning the men to the box via the engine. Responses were recorded and the general focus of attention (whether on the trips or on the men) was noted.

4.2.4 Number correspondence to circular motion

The materials for this task were a wooden merry-go-round with four seats and a number of wooden dolls (see Figure 8). Counters were used for money. After initial play the child was asked to choose dolls or one doll if preferred, to take a ride. One doll was identified by the child and he was asked to watch this one as it came around and wave every time it appeared in front of him. This was regarded as practice in identifying the full cycle. If the child could count he was asked how many 'turns' the doll had after each ride on the merry-go-round (1, 2 or 3 in successive trials). If the child could not count an alternative was to pay money, one coin or counter for each 'turn'. Records were made of responses and of the child's control of the toy, as well as the general focus of the child's attention.

An alternative toy was a wooden car with a black mark on one of its wheels. The child was asked to watch the mark go around as the toy was pushed. He had the opportunity to make the wheel go around as well as to count how often the tester made it turn in three trials, as with the merry-go-round. Recognition of one turn and control of the cycle of movement were the key behaviours (see Figure 9).

A note was made of which toy was used in the experiment in each case.

4.2.5 Conservation

Although this task is included in the analysis of number it was not administered at this point. It will be described in the following account of spatial tasks, in which context it took place.

4.3 Analysis

Correct responses were analysed for the effects of program differences by means of the MANOVA (Clyde, 1969).

4.4 Results and Discussion

While the MANOVA showed no significant program effects in the scores for number operations overall there is a great deal of individual variation in counting, number recognition and number prediction, and in the recognition of one cycle of a moving wheel (either carousel or car wheel). The majority of subjects could build a tower of matching height on level 'ground' (more in the post-test than in the pre-test) but few could build equal towers on different levels without considerable prompting. It is a characteristic of the pre-test that the testers gave somewhat more time to asking for reasons and provoking alternative responses than in the post-test, being more experienced in the underlying

concepts and in the clinical method. The majority of the scores for measurement in this difficult situation were reduced in the post-test. The task is one requiring conservation of length, a concept which Piaget believes (Piaget at al., 1960) is not acquired until after number correspondence and the conservation of quantity. It is not necessary for the child to use counting to solve the problem, but to recognise the compensations required by the displacement of the end points is essential. This is why the tester deliberately drew attention to the ends by pointing to the tunnel, the table top and then the highest points of the towers while demonstrating the test. A few subjects responded to the cognitive conflict provided by the inequality, for no matter how many blocks are used the towers could never look the same height. This forced those subjects to a consideration of the number of the blocks used in each tower, but the strategy was not always sufficient to produce a logical answer. Sometimes the subject miscounted in order to justify the original inequality.

The two number conservation tasks are included in this analysis because they test the subject's concept of the invariance of a number already quantified. In this case it is quantified by the one-to-one correspondence of copying a line of counters. Not surprisingly, most of our young subjects are influenced by the spatial arrangement of the counters, so that when the correspondence is destroyed, the number fails to be conserved. There is an increase in the numbers who conserve over the year and this can be attributed to development of a maturational sort as well as to experiences in the classroom. However, the conservers are differently distributed over sites in the post-test as compared to the pre-test, a fact which confirms the transience of

intuitions of logic at this age, and supports Carey's observations on concept mastery (cf. p. 29).

The post-test shows a general improvement in counting and in number recognition skills and in the ability to build towers of equal heights on equal surfaces. Number prediction does not improve to the same extent except in the Behaviourist Program. This shows that the mechanical skills are more easily acquired than the concepts. The need for perceptual supports for accuracy in quantification is emphasised in these results.

It is our belief that the strategies in handling the objects are of importance in gaining control over them and that the more able child makes a more flexible use of strategies. However, this experiment is a first attempt to find what the strategies might be. The tester had the difficult task of administering the tests and recording the results at the same time. In addition, there were administrative problems in some of the tasks. Some verbal instructions were difficult to comprehend. Some children, for instance, did not appear to understand the meaning of the word 'trip' (others interpreted a trip as one-way and this was accepted by the tester). Some of the toys, for instance the merry-go-round, although highly motivating, were also very distractive.

Consequently the data can be handled best in a descriptive manner. The NUCROS program (Macquarie, 1969) was used to draw up contingency tables and to provide chi square values for the nominal data. There were no significant associations between recorded strategies and correct responses. Some trends may be suggested by the numbers of responses falling into various categories of strategy or attentional focus.

Table 6 shows the numbers of different kinds of emphases and descriptions in some of the number and seriation tasks. Accuracy tends to be associated with the use of number more often than other descriptive content. It is clear, however, that the global configurations of colour or height are more attractive than the number of units and that the immediate task of filling and emptying the seats with little men, is more involving than the trips of the fire engine. The mathematical awareness exhibited by children who can use number or dimension descriptors to justify their judgements, or who can attend to one focus in the many changing aspects of each game, shows a gradual decentering of attention to perceptual features. The force of such observations is restricted in this experiment by the considerable degree of freedom allowed to the children and to the complexity of some of the games.

The impulsivity of some subjects makes it difficult to ensure that instructions are listened to and comprehended before the play begins. However, it is the control of impulsive behaviour which is associated with analytical style (Witkin, 1962; Zelniker and Oppenheimer, 1976) and which makes possible the structuring of the perceptual field. Some degree of control is exhibited, for instance, in the post-test results for number and motion correspondence. Table 7 shows the percentages of children who were correct in counting 1, 2 or 3 revolutions of the merry-go-round or the wheel. Part of the success in the post-test may be due to an increasing knowledge of number names, but it is more likely that the children have also learned to discriminate the beginning and end of one or more cycles more effectively. This trend was not evident in simple recognition of the cycle for which the criterion behaviour was waving, a continuous action.

Table 6: Action Emphases and Descriptions of Various Tasks

Task	Action Emphasis	N of responses		Description	N of responses	
		Pre	Post		Pre	Post
Seried beads	beads	13	9	colour	41	46
	height	13	27	dimension	20	31
Fire engine	trips	17	29	number	23	24
	men	23	52	action	19	32
Merry-go-round	movement	29	28			
	one doll	37	46			
Tower I	height	33	21	number	7	22
				verbal	20	29
Tower II	height	13	21	number	18	15
				verbal	14	13

Table 7: Percentages of Correct Quantification of Circular Movements

N of revolutions	Pre-test	Post-test
1	43.6	62.7
2	27.7	47.0
3	25.5	36.1
Spin control	41.5	67.5

Generally speaking the improved strategies are not powerful enough to deal with such matters as the estimation of the correct number of blocks to fit in a small box and the erection of an equivalent tower on an unequal surface. Improved accuracy in replacing the coloured beads (seriation) is associated with increasing reference to both colour and dimension in the child's description and with an action that focussed on the varying heights of the arrangements. Accuracy in building a tower on an equal surface is associated with production of a verbal explanation or counting the blocks.

In the division task the increased number of children who can predict how they might get the task done is associated with an increase of focus upon the trips and on the men, but a third of the children who focus on either make no prediction at all. The predictions of most children are not explicit. Few use sentences which reflect all of the actions and can be interpreted as number sentences. The majority of the explanations given after the event to justify the response to the question "How many trips?" incorporate actions but the actions do not necessarily correspond to the numbers given.

In the merry-go-round task more children focus upon the single element (the doll) in the post-test, and as this element marks the end of the cycle it is not surprising that there is a greater degree of accuracy in the post-test (see Table 7). There is an increase in the ability of the child to control the spin when he is playing with the toy (and to control the turn of the car wheel).

5. Spatial Perception and Spatial Intuitions

Spatial judgements are based on infralogical operations (Piaget and Inhelder, 1956), which require the support of imagery. Young children

rely on perception for much of their inference and the memory of a perception in the form of an image thus dominates spatial judgement. The quality of the image is dependent not only upon the quality of the perception but also on the cognitive structures available for its regularisation. As the perception becomes less global the child may be able to make judgements of similarity or difference on the basis of attributes, rather than on the overall shape and size of general configuration. Once the systematisation of exploration has begun, the child comes to rely less and less on perceptions as such and increasingly on systems of operations, or logical structures. However, these still require imagic support for the verification of concrete data. Even when the individual can work in the abstract mode of formal operations, Piaget and Inhelder claim he will require imagery for the support of spatial judgements.

At preschool age imagery is the source of spatial 'intuitions'. Hence some tasks which can be solved by logical means at a later age are presently solved by intuition. This is the case when the child cannot apply strategies of systematic search and verification. Out of the vast domain of spatial judgement were chosen two examples of ordering which could not be solved 'mathematically'. Order implies a set of relations between a number of objects and if these are not dimensional, they may be simple relations based on contiguity. Their memory requires the support of visual or other imagery.

When a standard is available the young child can match the order providing the distance between the copy and the model is small and the direction of the imitation is the same as the model's direction. It is extremely difficult to copy such an order in reverse (Piaget and Inhelder, 1956:81), because of the influence of the model upon the perceptions.

The influence of the perception also affects the quantification of the elements of the copy if it is arranged differently to the model. Thus a young child who has matched both the order and the number of a row of counterswill judge the number incorrectly in either the copy or the model if either is transformed in space. The reason is that the spatial configuration has been disturbed and the child has no strategies for quantitative evaluation of the situation. Similarly a spatial configuration cannot be reconstructed after rotation unless the memory image has incorporated some structural components of the configuration, the relations between the parts and the whole. It seems to Piaget and Inhelder that the child would need to incorporate actions into his imagery before he could use them to assist in reconstructions, particularly if the reconstruction required a different starting point in space. The experiences which would be vital for developing imagery of this sort are the actions and perceptions involved in the actual transformations.

When the child first attempts to determine the effects of a displacement in space the initial and the final configurations are taken into account but the intermediate steps are disregarded. However, some simple spatial tasks like predicting the order of a linear array, providing the number or elements is small, are within the capacities of young children even before transformational imagery occurs. If the image has recorded the original order the reverse may be intuited or reached by the processes of exclusion, providing no standard is present, for this will create perceptual interference in the re-ordering task.

The spatial rotations and reversals included in these mathematical games may be solved by such an understanding of the relations of contiguity between the elements. Since the objects are hidden during rotation, memory is required both for the individual items and their

ordinal relationships. In each case the child is required to make a prediction dependent upon his conservation of the linear order, but it is also possible for some predictions to be made on the basis of logical exclusion of the objects already in view.

5.1 Aims

The initial aim was to discover whether the children could copy a row of coloured plastic counters placed directly in front of them. Following this the equation of the two rows of counters was to serve as a basis for testing their conservation of the number of counters in one of the rows after displacement. The matching task was to serve as a preparation for the task ahead which was to discover whether the children could predict the linear order of a set of objects which they had selected and placed in a tunnel. First it was intended to reveal whether the order could be predicted in reverse, and then whether it could be retained after the tunnel had been rotated.

5.2 Method

There were in effect three tasks but only two sets of materials.

5.2.1 Spatial correspondence

Twenty plastic counters which can be divided into two equivalent colour sets and a wooden 'tunnel' were the only materials used in this experiment. The tester arranged ten counters in a row along one edge of the tunnel and the child was asked to copy this colour order along the opposite edge (about 4cm away). The tester said while making the row, "I want you to put your counters along there (pointing to the other edge) the same way as I am putting mine here. When I put one of this colour here you put one like it over there. Can you find a counter

the same as this one? ... Good, now this one". After the first two or three counters were copied the tester finished the row and then encouraged the child to finish in the same way. When the row was completed the child was given an opportunity to correct the response if necessary.

Scoring took place on the basis of proportions completed correctly (less than half, more than half, all) and whether corrections were made. Finally, the tester corrected any errors before going on to the next task of number conservation.

5.2.2 Number conservation

Using the same materials as before and with the arrangement of corresponding sets of counters, the tester asked the child to imagine that they were sweets. The ones nearest the tester belonged to her and the others to the child. She asked whether there were the same number in the following way, "Do we have the same amount; or do you have more; or do I have more?", pausing between each part of the question. After the child had agreed they were the same the tester removed the child's sweets from the tunnel edge and spread them out on the table. When they were arranged so that the space they occupied was about 60 percent greater than before the child was again asked the question above. He was asked to give a reason for his judgement and this was recorded. For the next spatial transformation the child's sweets were gathered together in a bunch at the front of the tunnel next to him and the question was put again.

Recognising that young children have difficulty with quantitative words, the tester was encouraged to put the question in different ways, using phrases like "as many" and finally, "Do we have the same? Those

are yours and these are mine, do we have the same? No? Who has more?"

5.2.3 Spatial inversion or rotation

Materials were four flat wooden blocks having the dimensions 2.5cm x 6cm x 4cm. These were covered on one side with a picture cut out from a magazine. There were two sets of these blocks.

Set A. a car, a flower, a round golden disc and some balloons;

Set B. a car, a motor bike, a boy, a plane.

The wooden tunnel already used (see Fig.10) served to conceal the blocks during the spatial transformation. The tunnel was just large enough for the blocks to be pushed in, so that when all four were in place one projected about one half a cm from one end of the tunnel.

The child was asked to name the pictures. Records were kept of the order chosen and the names given (the only one presenting any confusion was the disc, variously called money, a ball, a circle). The child was told to name each block as he placed them in the tunnel one by one.

a) Just as the last block was placed the tester put her hand over the other end of the tunnel and said "Tell me which one is going to come out first when you push. Tell me first and then I will move my hand away." The instruction was repeated in various ways until it was quite certain it was understood. After the prediction the tester moved her hand and said "Let's see if you are right". The tester then allowed the emerging block to be sighted and before returning it to the tunnel said "I am going to push this way now. Can you tell me which one is going to come out the other end?" After this prediction and return to the tunnel the direction was reversed twice more to allow the inner blocks to be named in their turn. Thus the order of exclusion of the four blocks was 1, 4, 2, 3 using the child's order of selection of the blocks.

b) When all of the blocks had been returned to the tunnel without disturbing their original order of placement, the child was told to watch while the tunnel was rotated 180°. The direction of pressure to exclude the blocks was now constant and the blocks were left out on the table as they emerged from the tunnel. The order of exclusion was 4, 3, 2, 1 and before each block emerged the child was asked to predict as before.

5.3 Analysis

The correct choices of each block were recorded and the data were analysed by multivariate analysis of variance for the effects of the different programs on the scores. Since there was a range of success on the various predictions the group mean scores (program by program) were ranked and a Friedman Two Way Analysis of ranks was performed in order to discover whether the eight different predictions were different from one another in order of difficulty.

5.4 Results and Discussion

5.4.1 Copying or matching a standard

This was not a very difficult task for the majority of the children who achieved a score of two or more (more than half copied correctly in both the pre-test and the post-test). With some exceptions group mean scores improved slightly in the post-test. On this occasion there was an all over effect of the programs on spatial ability ($p<.03$) but univariate statistics indicated that the variance did not occur in the matching task.

5.4.2 Predicting order after transformation

Since each prediction was a different task from the rest the scores were treated separately. The child could generally predict the end blocks but the inner blocks were more difficult especially in the reversal tasks, because the direction of emergence of the blocks changed after every trial. It must be conceded that the practice obtained by identifying each block as it emerged probably enhanced the accuracy of later trials. Thus the difficulty of the different predictions must be considered in relation to the sequence in which the trials occurred. It seems likely that the rotation trials might have been more difficult if they had occurred before the reversal trials. Such an assumption is in line with the research which suggests that rotation is more easily understood when the child is familiar with the stimulus materials (Hock and Ross, 1975).

Another important consideration is the salience of the blocks (Odom, Astor and Cunningham, 1975). The salience may derive in part from the picture on the block, and in part from the position of the block in the tunnel. Siegel (1972) found that the child's performance in a seriation task depends on both the length of the series and the relative position of an element in relation to the others.

In the post-test the predictions which showed the greatest variance program by program were the first and the second last. The former of these reached significance level of $p<.02$, the latter of $p<.064$ (not significant). This meant that the predictive achievements of children in some programs were enhanced in these trials, but the trials in question were not necessarily of similar difficulty. With training on the order of the blocks the first trial would have been very easy. However the child was not told in advance what the problem would be and therefore

had to make a prediction on the basis of recall of his first choice, providing of course he understood that the first block in would be the last block out.

The second last block prediction required much more than this. It required the child to have an image of the order of the four blocks, an image which would withstand rotation. The earlier reversal trials may have made it more difficult to establish the reciprocal relations between any two blocks because the direction from which predictions were made was changed after each trial. However, the exclusion of two blocks (4 and 3) which could be seen by the child may have assisted him in infering what the next one would be; and this would depend on his memory for the end block (Block 1), since this still remained in the tunnel. Thus two different information processes are involved - the memory image of the elements and the series and the reduction of the possibilities by exclusion. The salience of the pictures on each block may assist in the formation of images, but it may inhibit the memory of other factors such as the relations between the blocks as members of a series. One would expect that the first block chosen might have some special salience for the child and that this might at time enhance prediction, at others interfere with it.

The children in the Cognitive and the Contemporary programs achieved the highest mean scores with respect to prediction 1. However the children of the Contemporary program were not as successful in the post-test in prediction 7, the last but one, in spite of the fact that they had scored well in this trial in the pretest.

The mean scores of children in the five programs were tabled and ranked according to achievement on each of the eight prediction trials.

This permitted a Friedman Two Way Analysis of Ranks to be undertaken (see Tables 8 and 9). The differences between the eight sub-tasks are significant in the pre-test ($p<.05$). The column totals suggest that the most difficult sub-tasks were 3, 4 and 6. Prediction 3 required that the child recall the inner blocks and the relation of each to the end blocks. This is the first time the child is required to identify either of the inner blocks. In the following trial the remaining inner block is to be identified but the child may have some benefit from his success or his failure in prediction 3. The low ranking of mean scores in prediction 4 suggests that the child does not yet have an adequate image of the order of the blocks, and perhaps is not sure how many there are.

The only sub-task of comparable difficulty to these two is prediction 6, the first prediction of inner blocks after the rotation. Predictions 1 and 8 are the easiest since the means of these scores rank first and second. Predictions 2 and 7 are ranked together. Prediction 2 requires the recall of the end block which is the last to be chosen by the child, and therefore of low salience initially. Prediction 7 has been discussed (p. 46). Prediction 5 is of similar rank and is again the last block to be chosen by the child, but this time it is presented first after the rotation. It may assist the child to know that he has just seen this end block before the rotation, when the inner block (3) was predicted.

In the case of the post-test the difference between predictions is not significant. The general trend or pattern of the ranks is similar to the pre-test with one exception; prediction 6 appears to be the most difficult since the mean scores for this task rank the lowest. It is

Table 8: Rank Order of Mean Scores in Sub-tasks - Pre-test for Friedman Two-Way Analysis of Rank

Program	Sub-tasks 1	2	3	4	5	6	7	8
Cognitive	1.0	2.5	8.0	7.0	4.0	2.5	5.5	5.5
Competency	6.0	4.0	7.0	8.0	1.0	5.0	3.0	2.0
Contemporary	4.5	6.0	8.0	7.0	2.5	4.5	1.0	2.5
Behaviourist	1.0	3.0	8.0	7.0	4.5	6.0	4.5	2.0
Home Base	3.0	2.0	7.5	5.0	6.0	7.5	4.0	1.0
Totals	15.5	17.5	38.5	34.0	18.0	25.5	17.5	13.0

Table 9: Rank Order of Mean Scores in Sub-tasks - Post-test for Friedman Two-Way Analysis of Rank

Program	Sub-tasks 1	2	3	4	5	6	7	8
Cognitive	1.0	4.0	7.0	2.0	5.0	8.0	3.0	6.0
Competency	3.5	5.0	8.0	7.0	3.5	6.0	1.0	2.0
Contemporary	1.0	3.5	3.5	5.0	6.0	7.5	7.5	2.0
Behaviourist	5.5	3.0	5.5	4.0	2.0	8.0	7.0	1.0
Home Base	1.5	4.0	5.7	5.7	8.0	5.7	3.0	1.5
Totals	12.5	19.5	29.9	23.9	24.5	35.2	21.5	12.5

possible that some strategies are developing which alter the salience of the blocks in these tasks. Since the mean scores for prediction 4 are universally increased in the post-test, it is possible that this success (most probably by logical exclusion) interferes with the next task (6) involving the same block but approached from a different direction. Obviously the transitive relations between any two blocks had not developed to the level at which the child could say "Its next to the balloons this way and next to the money the other way".

The picture blocks are used for ease of naming by young children but it is obvious that the preferences for certain pictures made them more salient for ordering in some cases and in others interfered with the recognition of adjacent blocks. This depends on the kinds of strategies being used. That there are preferences is clear from a count of the first selections made by the children when asked to place one of the blocks into the tunnel. In Set A the car and the balloons are chosen more frequently than the others. In Set B the aeroplane is the favourite. Colour may be a less persuasive cue but at this age colour naming is inconsistent. Hence pictures are preferred as differentiators.

It is beyond the scope of this study to examine the intricate relationships between salience and success. What is indicated from these results is that discrimination training for the development of spatial relations should make use of salient features on which to build ideas of proximity and order in more than one direction. Then the salient feature can evoke images which are useful for ordering or positioning rather than distractive.

6. Summary and Conclusions

There were few consistent changes in the period from pre-test to post-test. However, some general increases in accuracy occurred but there were a few tasks in which scores were reduced for children in some programs. The post-test differs from the pre-test in that there is a greater variation between and within programs.

1. Equivalence classes: Subjects are more articulate in describing the classes they had formed using complete sentences with a superordinate structure, and finding more names for environmental materials. They tend to group objects in pairs in both the pre-tests and the post-tests unless prompted to add more by the experimenter.

2. Seriation: Both length discrimination and the arrangement of coloured beads in series had been mastered by most subjects in the post-test.

Many more children seriated spontaneously in the post-test, and since there was a significant relation between spontaneity of attack on the problem and its solution this indicated an improvement in the ability to arrange objects in series. However, the most difficult seriation task, the interweaving (series III), showed program differences in the post-test, thus indicating that the ability to break up a perceptual configuration and reform it in a new series, is susceptible to training or experiences of different kinds. Other program differences were found in the ability to identify the end points of a series, and in two of the three ordination tasks. The latter required the identification of ordinal correspondence in two series which were displaced and the program differences were in the selection of the inner blocks of the series.

3. Number: Counting and number recognition improved generally over the year, but the prediction of the number of blocks to fill an empty space did not. There were differences between post-test performances in other tasks contributing to the overall picture of inconsistency which has been found typical of children's thinking at this age. The number of children who could conserve number across transformations increased slightly and this is compatible with maturational change. The most interesting changes which were not statistically significant, were in the strategies of the subjects, in their more systematic approaches and the closer correspondence between number and unit of measurement.

4. Spatial transformations: The ability to copy spatial arrangements was greater in the post-test, and there were more successful predictions overall in the block task. However, the difficulty of some of the predictions remained and was increased in some instances by the interference of more cognitive strategies and their effects upon salience. There were significant program differences reflected in these scores.

The set of mathematical games required training the experimenters to administer tests in a games atmosphere and to keep the action going while recording intricate observations. Not surprisingly the reliability of the observations was affected by the stress upon the experimenter. In any case the clinical method made famous by Piaget lends itself to experimenter inconsistency unless the experimenters are highly trained in this art. However, these disadvantages are considerably offset by the formative possibilities of such evaluation procedures. The collection of observations of this kind over a number of different play problems can provide feedback about whether a particular emphasis in a curriculum is working in the way it was intended, or whether such

improvements as may be seen are acquired incidentally. These possibilities were not realised for individuals in different programs in the data collection year because the datawere not processed in time for changes in curriculum. Given that modifications are indicated in the experimental procedure to systematise observations and to restrict the options open to the subject the technique does offer both the teacher and the developmental psychologist an opportunity to assess attack strategies and their outcomes in the development of mastery of basic number concepts.

Figure 1

Figure 2

EQUIVALENCE

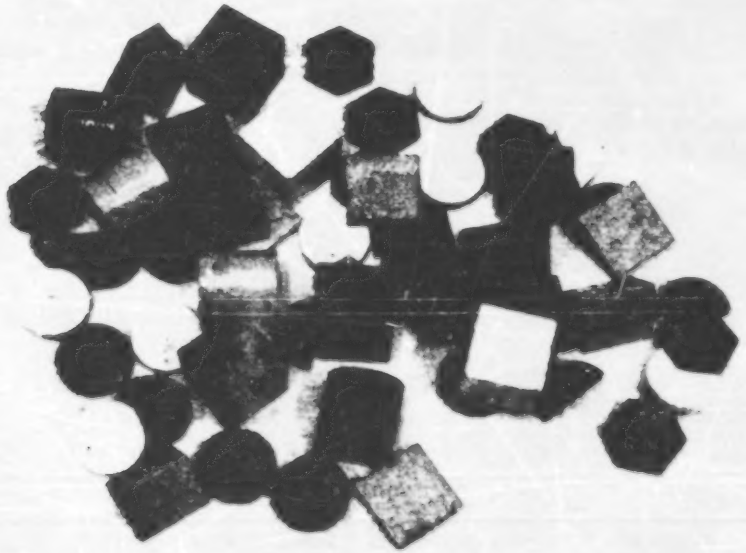


Figure 3

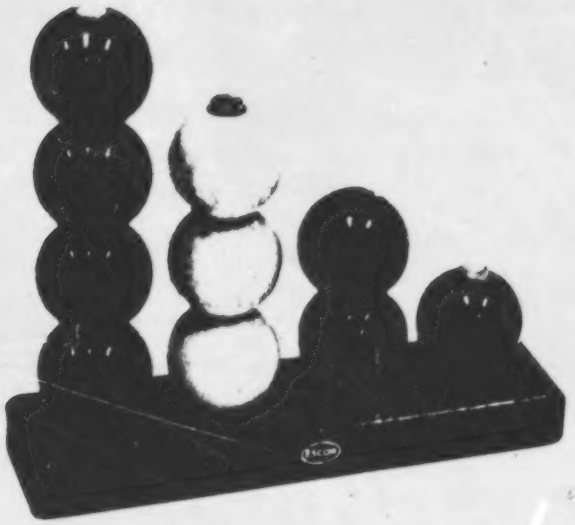


Figure 4

SERIATION

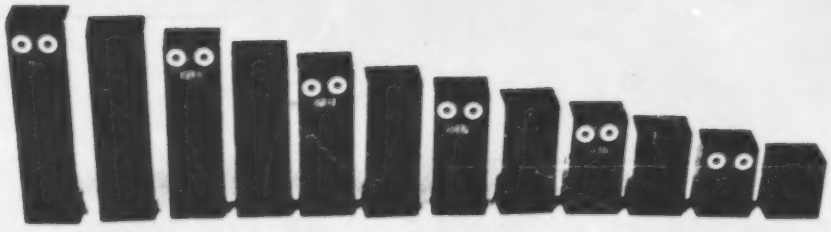


Figure 5

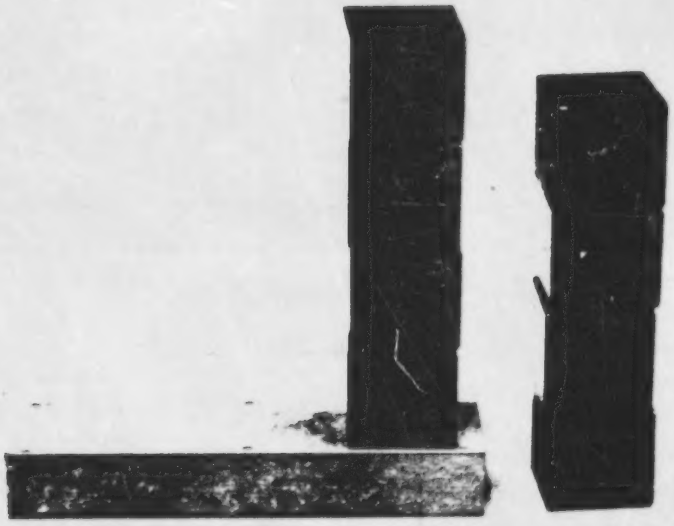


Figure 6

Figure 8

NUMBER

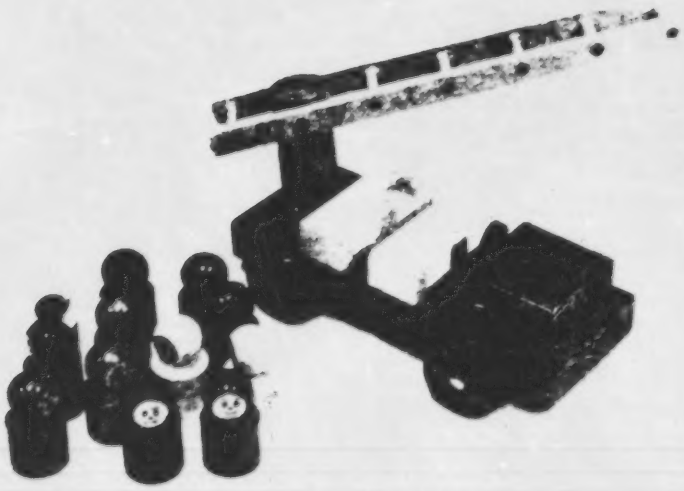


Figure 7

Figure 9

SPATIAL ROTATION

Figure 10

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Appendix A

MT. DRUITT EARLY CHILDHOOD PROJECT

MATHS RECORD

Date _____

Name _____ School _____

1 Equivalence	Items	Child's action	Description (verbal)
a) Toys			
1			
2			
3			
b) Objects			
1			
2			
3			
c) Blocks			
1			
2			
3			

Maths test

Name _____

2 Pretest Length

	Yes	No
Longest/Longer		
Same length/same size		
String As long as		

(Tick both whether correct and which term preferred)

3 Seried Balls

Verbal Description
1) colour
2) dimension or arrangement
3) other _____

Action accompanying description draws attention to
1) height
2) unit or number
3) other

Correct Yes No

Strategy of replacement
a. which end begins
b. mixed
c. stuck on incorrect
d. self corrects

4 Seriation

a) First series

	Yes	No
Order of blocks at finish: 1 ... 6 (chooses 1 and 6		
(spontaneous arrangement		
(provoked arrangement		
(incorrect arrangement		

b) Second series

	Yes	No
Order of blocks: 1 ... 6 (chooses 1 and 6		
(spontaneous arrangement after first two		
(provoked arrangement block by block		
(incorrect arrangement		

Verbal descriptions (e.g., recognises different heights of two series)

Recognition of difference Yes _____ No _____ Word used _____

Actions (general strategy)

	Yes	No
c) Double series: spontaneous after initial placing by E		
provoked block by block		
incorrect		
order ab reversed (ba)		

Order 1a 1b 6a 6b

Problems found

Maths test Name _____

4 Seriation (Cont'd.)

Ordination Record which block chosen	Yes	No	Comments
Relates 1a 1b			
6a 6b			
2a 2b			
5a 5b			
3a 3b			
4a 4b			

Measurement	Yes	No	Comments or Reason Given
a) Counts blocks correctly			
Predicts 2 to fill			
Recognises number - (6)			
Recognises number - (2)			
b) Builds same height tower			
Gives reason			
c) Builds same height tower			
Builts tower different size			
Gives reason			

Additional probes

Maths test

Name _____

5 Division (Fire engine)

Actions

Fills spaces only ____
Fills and adds more ____
Fails to remove men at fire ____
Fails to exhaust supply of men ____
Completes task ____
Self corrects ____

Number awareness (verbal)

Predicts N to fill seats ____
Suggests solution to getting all firemen there ____
Predicts N of trips ____
Says how many afterwards ____
On return counts 1 or more of the trips ____

Comments (Actions of child, words used, attention predominantly allocated to etc.)

6 Number and times correspondence

Carousel or wheel (underline which)

	Yes	No	How many?	Comment
Recognises one 'turn' (waves or nods)				
Counts one 'turn'				
two 'turns'				
three 'turns'				

Actions

Eyes (what attends to, e.g. 1 doll, movement of whole, etc.)

Hand (e.g., control of spin)

Maths test

Name _____

Spatial correspondence	Yes	No	Comment
a) Counters: Copies correctly			
Copies less than half correct			
Copies more than half correct			
Spontaneously corrects			
Refuses to correct			
b) Number conservation:			Reason
1. Spread out			
2. Bunched			

c) Blocks in tunnel — Name pictures

Order chosen 1. _____ 2. _____ 3. _____ 4. _____

		Yes	No	Comments
0^{o}	Predicts 1			
	Predicts 4 (reverse)			
	Predicts 2 (reverse of previous direction)			
	Predicts 3 (reverse of previous direction)			
180^{o}	Predicts 4			
	Predicts 3			
	Predicts 2			
	Predicts 1			